DETERMINATION OF HEAT-TRANSFER COEFFICIENT IN A BED OF SPHERES BY A QUASISTATIONARY CYCLIC METHOD

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Relations are presented for the amplitude ratio and the phase shift of the temperature of a gas in the periodic heating of a packed bed of spheres. The heat-transfer coefficient in the bed is determined by a cyclic method.

The determination of the heat-transfer coefficient in various types of heat-transfer beds is of particular interest. We consider one of the effective methods. If air enters a bed of spheres at the temperature

$$t(0,\ \tau) = t_Z + A_1 \cos \omega\tau, \tag{1}$$

its exit temperature is given by the expression

$$t(L,\ \tau) = t_Z + A_2 \cos(\omega\tau + \varphi_c). \tag{2}$$

The heat-transfer coefficient in the bed is determined either from the ratio A_1/A_2 of the entrance to exit amplitudes of the air temperature oscillations or from the main component φ of the phase shift φ_c. In exactly the same way as in [1], without taking account of the equivalent thermal conductivity of the bed, we obtain the relations

$$\ln \frac{A_1}{A_2} = \frac{\alpha F}{W} \frac{x(\mathrm{Bi}-2)S^+(x) + x^2C^+(x) - 2(\mathrm{Bi}-1)C^-(x)}{2x(\mathrm{Bi}-1)S^+(x) + x^2C^+(x) + 2(\mathrm{Bi}-1)^2C^-(x)}, \tag{3}$$

$$\varphi = -\frac{\alpha F}{W} \frac{x\,\mathrm{Bi}\,S^-(x)}{2x(\mathrm{Bi}-1)S^+(x) + x^2C^+(x) + 2(\mathrm{Bi}-1)^2C^-(x)}, \tag{4}$$

where $x = \sqrt{(2\omega/a)}R$. Some values of the functions

$$S^+(x) = \operatorname{sh} x + \sin x, \qquad S^-(x) = \operatorname{sh} x - \sin x,$$
$$C^+(x) = \operatorname{ch} x + \cos x, \qquad C^-(x) = \operatorname{ch} x - \cos x$$

are listed in Table 1.

TABLE 1. Some Values of the Functions S^+, S^-, C^+, C^-

x	$S^+(x)$	$S^-(x)$	$C^+(x)$	$C^-(x)$
0	0,000	0,000	2,000	0,000
0,1	0,200	0,000	2,000	0,010
0,2	0,400	0,002	2,000	0,040
0,4	0,800	0,021	2,002	0,160
0,6	1,201	0,072	2,011	0,360
0,8	1,605	0,171	2,034	0,641
1,0	2,016	0,334	2,083	1,003
1,5	3,127	1,132	2,423	2,282
2,0	4,536	2,718	3,346	4,178
2,5	6,649	5,452	5,331	6,933
3,0	10,16	9,877	9,078	11,06
4,0	26,53	28,05	26,65	27,96
5,0	73,24	75,16	74,48	73,92
6,0	201,7	202,0	202,6	200,7

It is shown in [1] that if the ratio A_1/A_2 is used to determine the heat-transfer coefficient, the most accurate results are obtained for small values of the dimensionless number $H = \alpha F / W_M \omega$. For H in the range from 0 to 0.2 Eq. (3), transformed to facilitate its practical application, is shown in Fig. 1. The curve for Bi = 0 is expressed by the equation

$$Z = \frac{W}{W_M \omega} \ln \frac{A_1}{A_2} = \frac{H}{1 + H^2}, \tag{5}$$

which can be used directly to determine H from the experimental data and thus to calculate the heat-transfer coefficient when the spheres have a high thermal conductivity. If, on the other hand, the thermal conductivity of the material of the spheres is low (Bi > 10 and Bi >

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TABLE 2. Characteristic Values and Conditions of the Experiments Performed

t_Z, °C	A_1, °C	$\frac{A_1}{A_2}$	ω, 1/sec	$\frac{\alpha F}{W}$
40,0—70,0	10,0—20,0	2,5—8,5	1,1—2,1	1,0—2,5
H	Bi	m	$\frac{L}{d_S}$	Re
0,04—0,15	0,04—0,20	0,385—0,430	4,5—9,0	2500—10000

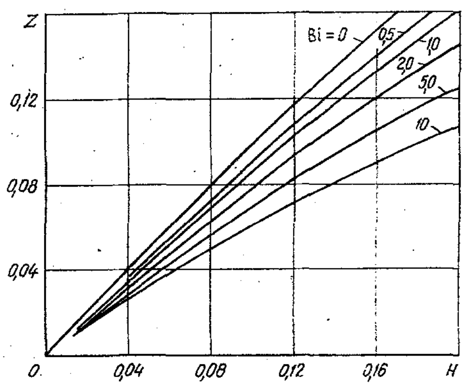

Fig. 1. Z = f(H, Bi) for a bed of spheres

50H) the ratio of the amplitudes of the temperature oscillations and the main component of the phase shift are given by the relations

$$\ln\frac{A_1}{A_2}=\frac{\alpha F}{W}\cdot\frac{1+\frac{\alpha F}{W}\varkappa}{1+2\frac{\alpha F}{W}\varkappa+2\left(\frac{\alpha F}{W}\varkappa\right)^2},\qquad(6)$$

$$\varphi=-\frac{\alpha F}{W}\cdot\frac{\frac{\alpha F}{W}\varkappa}{1+2\frac{\alpha F}{W}\varkappa+2\left(\frac{\alpha F}{W}\varkappa\right)^2},\qquad(7)$$

(where $\varkappa = (W/F)/\sqrt{2\lambda_M\rho_M c_M\omega}$) which follow from the solution of the periodic heating of a semi-infinite body. The heat-transfer coefficient can also be determined directly from Eqs. (6) and (7) by using the dimensionless parameter $\alpha F/W$.

The average heat-transfer coefficient for the flow of air through a stationary packed bed of spheres was determined experimentally by the procedure described above. Steel and corundum spheres 10 and 15 mm in diameter were placed in a thermally insulated thin-walled channel of rectangular cross section. The channel was 80 mm wide and the thickness of the bed varied from L = 60 mm to L = 90 mm. The range of basic quantities characterizing the experiments performed is shown in Table 2. The temperatures before and after the bed were measured by a network of thermocouples made of 0.1-mm-diameter wire placed in the central part of the channel outside the distribution grids holding the bed of particles. The effect of the grids and the channel walls on the deviation of the temperature oscillations was established by investigating the amplitude and phase characteristics of the experimental channel without the bed. The ratio of the amplitude of the actual temperature of the flowing air to the amplitude shown by the measuring device

$$\frac{A}{A_T}=\sqrt{1+(\omega C)^2}$$

depends only on the time constant C of the thermocouples and the measuring device and on the inner frequency ω of the temperature oscillations. If the measuring device placed before and after the bed has the same values of C, as was the case in our experiments, the readings do not have to be corrected for the cyclic method. The temperature oscillations were produced by mixing hot and cold air in such a way that the temperature of the air entering the bed did not differ from the value given by Eq. (1) by more than 2%. The pressure in the mixing device was regulated to ensure a constant flow rate of air during periods of heating and cooling. The flow rate, determined from the total discharge measured by a diaphragm directly behind the bed, was computed for the central part of the experimental channel; the change in porosity of the bed in the edge region close to the channel walls was taken into account [2, 3].

Since the porosities of the various beds varied over narrow limits (m = 0.385-0.43) the experimental results can be represented by a relation between the dimensionless numbers $Nu = \alpha d_S/\lambda$ and $Re = wd_S/\nu$. The values of Nu calculated from Eq. (3) are shown as a function of Re in Fig. 2. For comparison the figure also shows points representing earlier data processed by Timofeev [4], and results from [5], [6], and [7] also obtained by a cyclic method. The shaded region shows the range of experimental values of Nu reported by a number of authors [8]. From our data the heat-transfer coefficient for the flow of air through a stationary packed bed of spheres in the range of Re from 100 to 10,000 can be approximated by the equation

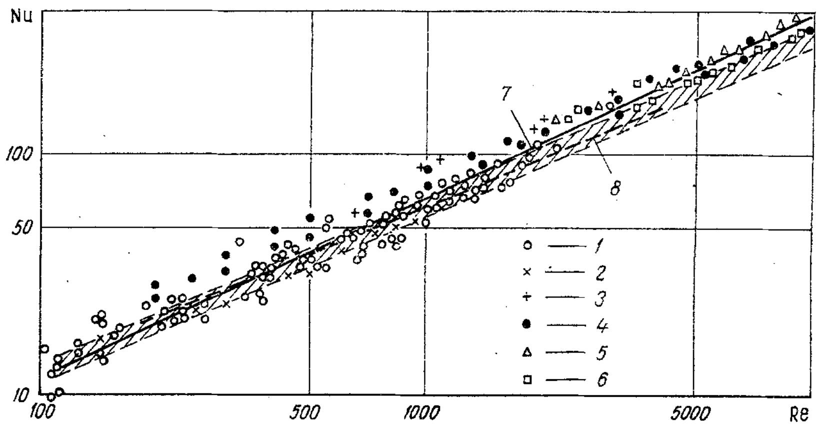

Fig. 2. Nusselt number as a function of the Reynolds number for a stationary packed bed of spheres. Data from: 1) [4]; 2) [5]; 3) [7]; 4) [6]; the shaded region represents data from [8]; 5, 6) our data for steel and Al_2O_3 spheres,respectively; 7) $Nu = 0.33\ Re^{0.77}$; 8) $Nu = 0.61\ Re^{0.67}$ [4].].

$$Nu = 0.33\,Re^{0.77}$$

with about the same accuracy as in [4] over the narrower limits of definition.

NOTATION

A_1, A_2	are the amplitudes of temperature oscillations of gas at entrance and exit;
A_T	is the amplitude measured by thermocouple;
a	is the thermal diffusivity;
$Bi = \alpha R/\lambda_M$	is the Biot number;
c_M	is the specific heat of material of particles;
d_S	is the diameter of spheres;
F	is the heat-transfer surface of bed;
$H = \alpha F/W_M\omega$	is a dimensionless number;
L	is the length of bed;
m	is the porosity of bed;
$Nu = \alpha d_S/\lambda$	is the Nusselt number;
R	is the radius of particles;
$Re = wd_S/\nu$	is the Reynolds number;
t, t_Z	are the instantaneous and average temperatures of gas at entrance;
w	is the permeation velocity of gas;
W, W_M	are the water equivalents of gas and bed;
α	is the heat-transfer coefficient;
λ, λ_M	are the thermal conductivities of gas and particles;
ν	is the kinematic viscosity;
τ	is the time;
φ_C, φ	are the total phase shift and its main component;
ω	is the angular frequency of temperature oscillations.

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